NEW MATHEMATICS

Understanding SETS

J. Kennedy

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Understanding Sets

THE SERIES
Understanding Sets
Understanding Number
Understanding Shapes and Solids
Understanding Graphs and Statistics

NEW MATHEMATICS

Understanding Sets

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Note to teachers

I have written these notes and exercises in the hope that they will provide a simple introduction to the mathematical notion of Set. I feel that much of the work outlined will be within the compass of children in the top classes of primary schools. It is hoped that the experiences provided will (1) help children to an understanding of one of the most important and underlying concepts in mathematics, and (2) provide them with some new language which they may later employ to express ideas in arithmetic, geometry and algebra with greater accuracy and precision.

The concept of a set is to be found at the very heart of almost all topics in modern mathematics. Set thinking permeates most areas of mathematics from the basic arithmetic of numbers to the advanced and theoretical abstract topics of research mathematics. Developing a sound understanding of what a set is, how sets are related, and how to operate on sets can lay a firm foundation for the more abstract ideas that the pupil will meet in his future study of contemporary mathematics.

The science of sets tells us how to combine sets and how to compare sets to determine relationships. Solving equations, drawing graphs, studying chance or probability, describing geometrical figures become simpler by using the language and ideas of sets. Particularly exciting is the use of sets in solving problems, puzzles and mysteries with electronic computers.

Children have the notion of set from quite an early age. When they collect toys or group things they are acquiring their first notion of sets. When they collect or place together two sets of objects they are performing what is known in mathematics as a *union of sets*. Union is an operation on sets in much the same way as addition is an operation on numbers.

The union of two concrete sets is fundamental to the understanding of the arithmetic operation of adding numbers. Numbers are highly abstract, and more difficult to understand than the physical members of a concrete set.

In mathematics programmes in the USA the notion of set is developed very early to help children distinguish between numerals and numbers.

The programmes make no attempt to define the term 'set'. 'It is impossible to define every idea in terms of simpler ones: it is necessary to start with certain undefined terms, and the idea of set is so intuitive and basic that it is a convenient starting place'.

'Set' can be taken as synonymous with such words as collection, group, team, class, club, herd, flock, etc. A set, then, is any collection or group of things or ideas which is described accurately enough to tell whether or not a particular thing belongs to it. It may be a collection of pictures, persons, ideas, lines, numbers, colours, etc.

Objects are not sets of themselves. They become a set only when we *think* of them as belonging to a set. The basic assumption is made that a *set* is determined by its *members* (or *elements*). The collection of objects must be conceived *as a whole.*

The basic concept is *membership.*

Children think about many sets. There is a very early awareness that things belong in pairs together, as shoes, gloves, hands, feet, sleeves, etc. They collect bricks and beads and sort them according to colour and size. They make collections and count them. There is the family group and its correspondences in laying the table, etc.

Grouping is a fundamental part of primary school arithmetic. The four operations of addition, subtraction, multiplication and division are purely processes of regrouping : the coming together or separation of groups into new groups.

The preliminary work will consist of collection of groups or sets by the children. This is an exercise simply to emphasize the notion of a set being a collection.

Once this notion is established the work will transfer to the main purpose of these exercises : the collection of *specific* sets. Children must learn that when they name sets they must be explicit so that only one specific set is appropriate, e.g. *a* set of numbers is not a specific set, but *the* set of numbers beginning with 4, and counting by 2 to 12, is a specific set.

The work will then proceed to the observation of set relationships and operations, and the incidental acquisition of the mathematical language we use to describe these relationships and operations.

Sets and Set Language

SETS - THINKING ABOUT COLLECTIONS OF THINGS

1 Make a collection from magazines, newspapers, catalogues, books, etc., of pictures of groups of things.

At first keep to objects of the one kind, as a flock of birds, a flight of aeroplanes, a group of boats, flowers, men, women, children, footballers (from the sports pages of newspapers), hats, coats, horses, cows, cars, books, and so on.

2 When you have got a really good collection make a class Set Book of them.

Paste them into your Set Book and label each one.

You might label them : 'This is a *collection* of boats' or, better still, 'This is a *set* of boats'.

Some sets have special names like pack, bunch, regiment, swarm. These are names for sets of things usually found together like a pack of cards, or a bunch of flowers.

There are other sets that have no special name because they contain things that are not often grouped together, e.g. the set of objects in your pocket, the contents of your school bag, the objects in lost property in your school. If you wrote the items in these sets you would get an odd assortment !

Can you write down in your set book :

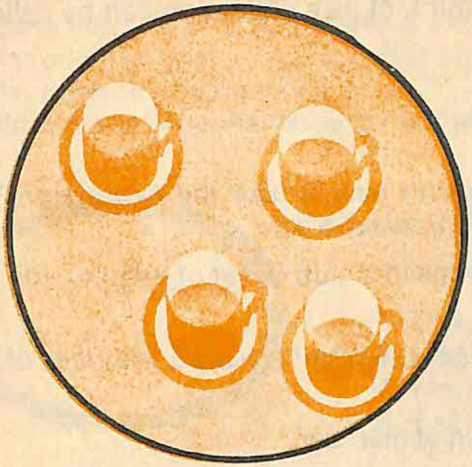
1 A set of five things you can buy at a grocer's
2 A set of four things you can buy at a butcher's
3 A set of six living things
4 A set of three boys' names
5 A set of five whole numbers
6 A set of four flowers

In mathematics we have to learn short ways of writing things down. It saves a great deal of time. You already know a few 'shorthand' ways of writing things down : =, +, −, ×, ÷.

You are going to learn quite a number of new 'shorthand' signs in this book.

3 Now make some drawings showing sets of things.

Put a circle or a ring round them so as to shut in the things you have drawn and we can see quite clearly what things belong to your set. Like this:



And don't forget to label underneath:
'This is a set of cups', or whatever it is.

4 Make some more collections, this time of sets of mixed objects.

Label your collections like this:

'This is a set of boys and girls' or 'This is a set of children'.

'This is a set of cows and sheep' or 'This is a set of animals'.

We can make the label shorter: 'A set of toys', 'A set of cutlery', and so on.

Here is one:

A set of crockery **A set of crockery**

So now you know something about sets. You know that a set is a collection — *like a flock, a team, a group, a class, and so on.*

The things of a set may be objects, people, letters of the alphabet, colours, names, numbers, etc. Anything you can think of as a collection can be called a set.

MEMBERS OF SETS

Perhaps I should tell you here that we don't usually use the expression 'the things of a set'. We usually speak of the '*members* of a set'.

It is a better word really. If your father belongs to a club (a set of people) you say he is a 'member' of that club.

If you are in the school football team (also a set) we say you are a '*member*' of the team.

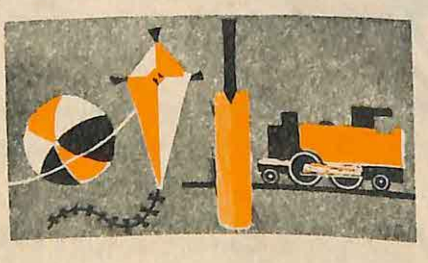
Some people use the word 'element' instead of member.

The drawing shows the members of the Brown family. It can be called the *set* of the Brown family. The *members* of the set are the father, the mother, the daughter and the son.

Exercise 1

Answer these questions:

1 How many members are there in the set of the Brown family?
2 How many grown-up members are there in the set?
3 How many child members are there?
4 Is Grandma Brown a member of this set?

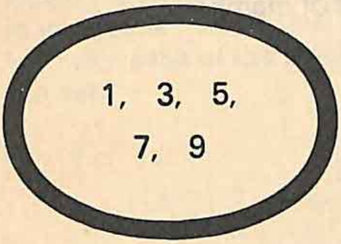


Here is a *set* of toys

Exercise 2

Complete these sentences :

1 There are.......... members of this ....
2 The engine is a m.......... of this set.
3 The other members of the set are ....
4 We call anything that belongs to a set a ....



5 This is a set of ....
6 There are five .... of this set.
7 The figure 3 is a .... of the ....
8 The other members are ....

Exercise 3

Write the following sets. Remember to enclose them in a circle or ring :

1 A set of letters with six members.

2 A set of even numbers with five members.

3 A set of straight lines with four members.

4 A set of boys' names with three members.

5 A set of six odd numbers.

6 The set of the days of the week.

7 The set of the first six months of the year.

8 A set of girls' names with five members.

9 The set of odd numbers less than ten.

10 The set of months beginning with M.

11 The set of numbers on a clock face.

12 The set of numbers on the calendar for June.

13 The set of names of children in your class who are Brownies.

14 The set of names of children in your class who are Cubs.

Answer these questions:

15 How many members has the set of classes in your school?

16 If we wrote down the set of children in your class how many members would there be?

17 How many members in the set of children in the school?

18 Can you find out how many members there are in the set of people in your town?

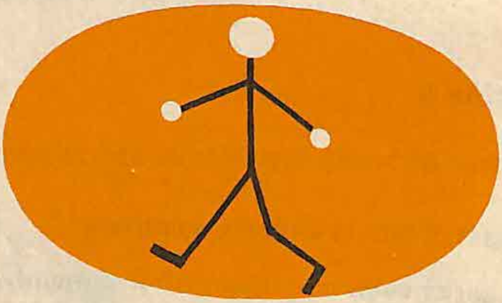
19 Can you think of sets with an even bigger number of members?

Some sets have a very big number of members.

A set may be made up of any number of members.

Exercise 4

Let us look at some more sets:



1 How many members does this set have?

2 Write down the set of months whose names begin with the letter F. How many members has it?

3 How many days of the week begin with the letter M?

4 How many members in the set of days whose names begin with the letter M?

5 How many even numbers between 7 and 9?
Write the set of even numbers occurring between 7 and 9. How many members in the set?

6 How many odd numbers between 4 and 6?
Write the set of odd numbers between 4 and 6. How many members in the set?

So we find that there are some sets with only *one* member.

Collect some pictures of sets which have only one member.

This next part is very important. Try hard to understand it properly.

We must make quite clear now the difference between 'a set of things' and 'the set of things'.

You see, if I say '*a* set of books' you are not at all clear about what books are meant. I could be meaning any set of books at all.

But, if I say '*the* set of books on the top shelf of the bookcase', or '*the* set of books in your desk', you know exactly which books I mean. You could, if you wanted to, make a list of the books. You could write down the names of the members of the set.

This is the kind of set we are going to deal with: the set whose members we know, or can find out.

The members may be named in two ways:

(i) by making a list of the members (ii) by describing the members,

e.g. (i) by listing: 11, 13, 15, 17, 19.
(ii) by describing: the odd numbers between 10 and 20.

or, (i) a, e, i, o, u.
(ii) the set of vowels.

or, (i) 2, 4, 6, 8, 10.
(ii) the even numbers from 2 to 10.

When we say 'the set of ....' the way we describe the set should tell us what items belong to the set and what items are not members of the set.

The objects (or members) of the set must be looked on *as a whole.* When the grocer brings a parcel of groceries to your house the parcel is the *set* of all objects in the parcel. When we open the parcel, it isn't a parcel or a set any longer.

If there was only one thing in the parcel (say a leg of ham) it would be a set with only one member.

The following little exercises will help you to form a good idea of definite (or specific) sets. For the moment, just do as your teacher asks as he reads out each command. (Later on, perhaps you will write out these sets and put them in your Class Set Book. Two or three children could undertake the job of collecting one of the sets.)

1 Stand up the boys in the class.
2 Stand up the girls in the class.
3 Stand up the children with dark hair.
4 Stand up the children with fair hair.
5 Stand up the children who have brothers.
6 Stand up the children who have sisters.
7 Stand up the children who have brothers and sisters.
8 Stand up the children who have neither brothers nor sisters.
9 Stand up the children who have their own bicycles.
10 Stand up the children who live in .... Road.
11 Stand up the children who live in .... Street.
12 Stand up the children who have a dog pet.
13 Stand up the children who have guitars.

All these are sets whose members we can list and about which we can say 'so and so is not a member of that set'.

NAMING SETS

We have collected quite a few sets already including sets of birds, footballers, toys, cricketers, dishes, etc.

In mathematics we usually look out for short ways of writing things.

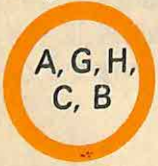
For example: for a set of birds we might write Set B.
for a set of footballers we might write Set F.
for a set of toys we might write Set T.
for a set of cricketers we might write Set C.
for a set of dishes we might write Set D.
and so on.

We do not, however, have to use the first letter of the name of the things in the set. We could call the set of birds Set A or Set X or any other letter.

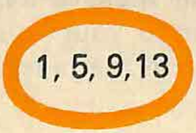
WRITING DOWN SETS

There are several ways of writing down sets. You have learned one way already, that of putting them in a circle or a ring.

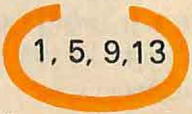
Like this:



or this:



But not like this:



because the circle is not complete, and the members are not properly enclosed.

Another way of writing sets, the most usual way, is to put brackets round the set like this:

Set X = {A G, H, C, B} Set Y = {1, 5, 9, 13} Set Z = {6, 8, 10, 12}

Let us have some practice now in writing down sets in this way.

Question 1 Write the set of the first five letters of the alphabet.
Answer **Set A = {a, b, c, d, e}**

Question 2 Write the set of the numbers you use when you count the first five children in your class.
Answer **Set N = {1, 2, 3, 4, 5}**

Question 3 Write down the set of the days of the week.
Answer **Set D = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}**

Question 4 Write the set of months of the year whose names begin with letter J.
Answer **Set M = {January, June, July}**

Question 5 Write the set of odd numbers between 0 and 10.
Answer **Set O = {1, 3, 5, 7, 9}**

Now it is your turn. Don't forget the brackets and the commas.

(The expression 'from a number to another number' includes both numbers, e.g. from 11 to 15 means "*11*, 12, 13, 14, *15*.")

Exercise 5

Write the members of each of these sets:

1 The last four letters in the alphabet.
2 The days of the week whose names begin with T.
3 The numbers counting by 2 beginning with 6 and ending with 16.
4 The set of odd numbers beginning with 11 and ending with 19.
5 The set of even numbers less than 12.
6 The set of whole numbers less than 17 and larger than 15.
7 The set of counting numbers from 10 to 17.
8 The set of counting numbers from 51 to 60.
9 The set of counting numbers from 110 to 115.
10 The set of counting numbers from 697 to 703.
11 The set of vowels.
12 The set of days in the week whose names begin with letter S.
13 The set of names of the children in your family.
14 Give me the names of five sets of which *you* are a member.
15 All the factors of 12.

16 The multiples of 5 that are less than 36.
17 All multiples of 7 that are odd and less than 50.
18 All factors of 20.

Can you answer these questions about sets?

Exercise 6

1 How many members are there in the set of letters of our alphabet?

2 How many members in the set of vowels in the alphabet?

3 Write the set of letters in your first name. (If a letter comes more than once in your name you need to put it down only once in your set.)

4 How many members has the set of the months of the year?

5 How many members has the set of days of the week?

6 Write the set of the first letters of the names of the months of the year without repeating any letter.

7 Write the set of the numbers between 10 and 20 that can be shared exactly by 3.

8 Write the set of whole numbers less than 20 that can be divided exactly by 4.

Exercise 7

Here are some sets. Below each set are groups of words. Which *best* describes each set? Is it (a), (b), or (c)?

Write your answer. Then tell how many *members* there are in that set.

1 {1, 3, 5, 7, 9, 11}

(a) a set of small numbers
(b) the set of all odd numbers
(c) the set of odd numbers less than 12

2 {Saturday, Sunday}

(a) the set of school days
(b) the set of the first two days of the week
(c) the set of days in the week whose names begin with S

3 {10, 20, 30, 40, 50}

(a) the set of numbers less than 60
(b) the counting numbers less than 60 whose numbers end in nought
(c) the set of even numbers less than 60

4 {chalk, book, rubber, pencil}

(a) a set of things you find in a schoolroom
(b) a set of school furniture
(c) a set of things to read

In the next exercise you are given the set to describe as well as you can. Here are one or two examples for you:

Set A = {1, 2, 3, 4}

Set A is the set of the first four counting numbers.

Set B = {a, b, c, d}

Set B is the set of the first four letters of the alphabet.

Set C = {9, 12, 15, 18}

Set C is the set of numbers from 9 to 18 that are divisible by 3.

Now try to describe these sets:

Exercise 8

1 Set D = {w, x, y, z}

2 Set E = {1, 3, 5, 7, 9}

3 Set F = {January, June, July}

4 Set G = {10, 12, 14, 16, 18, 20}

5 Set H = {100, 200, 300}

6 Set I = {5, 10, 15, 20, 25}

7 Set J = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

8 Set K = {V, VI, VII, VIII, IX, X}

9 Set L = {a, e, i, o, u}

10 Set M = { ■, ◢, ⏢, ◆ }

11 Set N = {Tuesday, Thursday}

12 Set X = {pen, book, paper}

13 Set Y = { car, bus, van}

14 Set Z = {12, 16, 20, 24, 28}

EMPTY SETS

If you look back to the earlier part of this book you will find (Page 6) that we could collect sets of all sizes, large and small. Look at Exercise 4 (Page 6).

In this exercise you collected quite a number of sets *with only one member.*

There are some sets with *no members at all.*

How many members are there in this set?

How many children in your class are twenty years old?

How many members are there in the set of children in your class who are twenty years old?

Make a drawing of the set of children in your class who are twenty years old.

How many members are there in the set of months whose names begin with letter B?

Make a drawing to show the set of months whose names begin with letter B.

How many children in your class are under 50 centimetres tall?

Make a drawing of this set.

You will have noticed in each of these examples that the drawing or picture is empty.

If a set has no members we call it an *empty set.*

Some people call it a *null set* which means the same thing.

If we use brackets we write Set A = { } which means that Set A has no members and is an empty set.

We have another way to show the empty set. It is to use the sign or symbol ø.

So, if a set is an empty one, we can write it down as:

Set A = { } or Set A = ø

The two symbols { } and ø mean the same thing.

Here are some examples of empty sets:

1 The set of children in your class who are less than four years old.
2 The set of letters of the alphabet between D and E.
3 The set of triangles with four sides.
4 The set of numbers between 5 and 6.
5 The set of numbers between 11 and 19 which divide by 10.
6 The set of numbers between 30 and 40 which are larger than 50.
7 The set of children in your class who have beards.

There are very many empty sets.
See how many more your class can collect.
Put them in your class book and label them properly: the set of . . . = { } or ø.

MEMBERSHIP

Now before we finish with this chapter describing sets and their members, there is more shorthand (two symbols) which you should learn.

∈ stands for 'is a member of'

It is rather like a letter E. Do you remember that we once said a member is sometimes called an 'element'?

So, if we say b is a member of Set {a, b, c, d},
we could write b ∈ {a, b, c, d}

The other sign is ∉. That is, ∈ crossed out with a slanting line through it.

Can you guess what it means? Did you guess right? It means 'is *not* a member of'.

Thus, we would say g ∉ {a, b, c, d}
i.e. g is not a member of {a, b, c, d}.

To get into the way of using these signs do this next exercise. The *signs* meaning either 'is a member of' or 'is not a member of', have been left out. You have to put the correct sign on the dotted line. Do them in your own note-book.

Exercise 9

1 3 . . . . {the odd numbers}

2 Blue . . . . {red, white and blue}

3 14 . . . . . . {prime numbers}

4 Sunday . . {set of days in the week}

5 Tom. . . . . . {girls' names}

6 Brian . . . . {boys' names}

7 d . . . . {set of vowels}

8 Wednesday {days of week}

9 Thursday . . {months of year}

10 12 . . . . . . {multiples of 4}

11 4 . . . . {factors of 18}

12 81 . . . . {multiples of 3}

13 51 ...... {prime numbers}

14 32 ...... {multiples of 6}

15 16 .... {square numbers}

16 29 .... {prime numbers}

Now try to make up six examples of your own, showing how to write down in a short way that a thing is, or is not, a member of a certain set.

Exercise 10

Fill in the blanks.

1 Mini-minor $\in$ ....

2 Bread $\in$ ....

3 Coal $\in$ ....

4 Carrot $\in$ ....

5 .... $\in$ {flowers}

6 .... $\in$ {birds}

7 .... $\notin$ {trees}

8 .... $\notin$ {games}

Revision

Now before we pass on to something new let us sum up what we have learned so far about sets.

We can call any collection of things a *set*.

The things that belong to a set are called its *members*.

A set may have a lot of members, a few members, one member, or no members at all.

If it has no members we call it an *empty* (or *null*) set.

When we make a drawing of a set we enclose it in a circle or ring.

If we write a set we put the members in brackets like this: {1, 3, 5, 7}.

We show an empty set like this { } or ø.

The sign $\in$ means 'is a member of'.

The sign $\notin$ means 'is not a member of'.

If you have remembered all these things you are ready to learn something more about sets in the next chapter.

Comparing Sets

Look at these pairs of sets.

{1, 3, 5, 7}   {1, 3, 5, 7}

{a, g, e}   {a, g, e}

{May, June, July}   {May, June, July}

{▲, ▮, ◓, ■}   {▲, ▮, ◓, ■}

What do you notice about them?

You have probably noticed that each pair of sets has exactly the same members.

We say then that these sets are equal.

Now look at these pairs:

{1, 3, 5, 7}   {3, 5, 1, 7}

{a, g, e}   {a, e, g}

{May, June, July}   {May, July, June}

{▲, ▮, ◓, ■}   {▲, ◓, ▮, ■}

What do you notice about them?

Yes, they have *the same members,* though not this time in the same order.

We can say these sets *are equal* (*or identical*).

Some people use this word identical which means 'equal' or 'the same'.

This then is one of the ways in which we compare sets. We compare the members of one set with those of another.

If they have the same members we say the sets are equal.

And don't forget, it doesn't make any difference whether the members are in the same order or not.

Set P = {b, c, d}   Set M = {c, d, b}

then   Set P = Set M

or   P = M

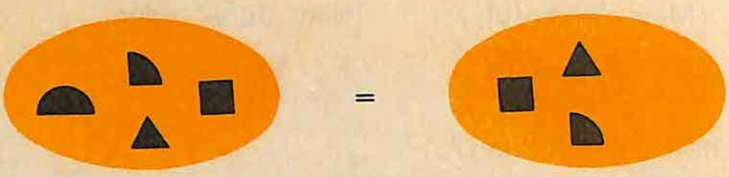
Exercise 11

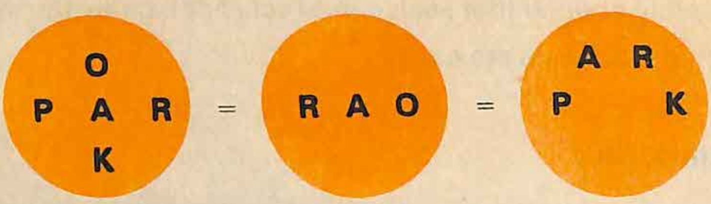
Complete the following sets to make them *identical* or *equal.*

{5, 10, 15, 20} = {10, ..., 20, 15}

{7, 11, 15, 19} = {11, 19, ..., 7}

{A, E, M, T} = {M, A, ..., ...}





Collect some pictures giving different views of the same group. (Two different newspapers will provide these.) When you have a good collection put them in your book and label them *identical* or *equal sets.*

We say that sets are identical when ....

If Set A and Set B are identical or equal we write:

A = B

Exercise 12

Look at these sets and answer the questions underneath them.

A = {6, 8, 9} B = {8, 6, 9} C = {9, 8, 6}

1 Does Set A = Set B?
2 Does Set C = Set A?
3 Does Set C = Set B?

X = {r, o, c, k} Y = {c, o, k, a} Z = {k, c, o, r}

4 Does Set X = Set Y?
5 Does Set Z = Set X?
6 Does Set Z = Set Y?

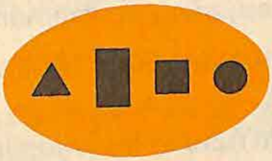
Now look at these sets:

(a) Set M = {5, 10, 15, 20} Set N = {10, 20, 15, 25}

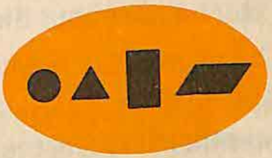
(b) Set D = {7, 11, 15, 19} Set F = {17, 15, 11, 9}

(c) Set G = {A, E, M, T} Set H = {M, E, H, T}

(d) Set J =



Set K =



Are any of the above pairs of sets equal?

If two sets are not equal, we cannot say, for example, Set X = Set Y. We use this sign ≠ (Can you guess its meaning?), and we say:

Set X ≠ Set Y

(Set X is not equal to Set Y)

If then we compared the sets (a) to (d) above, we would write:

(a) Set M ≠ Set N

(b) ....

(c) ....

(d) ....

You finish them.

Exercise 13

Put in the correct sign (= or ≠) between the following sets:

1 {3, 13, 23, 33} .... {13, 23, 33, 43}

2 {A, C, E, G} .... {G, C, A, E}

3 {Mary, Tom, Bill, Joan} .... {Bill, Mary, Joan, Tom}

4 {peas, beans, cabbages} ...... {cabbages, beans, potatoes}

Make up some sets of your own that are not identical and write them down *using the proper sign.*

Sets within Sets

Look at these two sets : Set X = {A, C, E, G, I}   Set Y = {E, G, I}

Read aloud the members of Set X. Read aloud the members of Set Y.

What do you notice about them ? Have they anything in common ?

Yes, you have probably noticed that the members of Set Y (E, G and I) are also to be found in Set X.

To put it another way : all the members of Set Y are to be found in Set X.

Write down what you notice about these two sets.

1 Set A = {1, 3, 5, 7, 9, 11}   Set B = {5, 7}
2 Set A = {red, white}   Set B = {brown, blue, red, white}
3 Set A = {a, b, c, d, e}   Set B = {a, e}

You will have noticed again that all the members of one of the sets are contained in the other set.

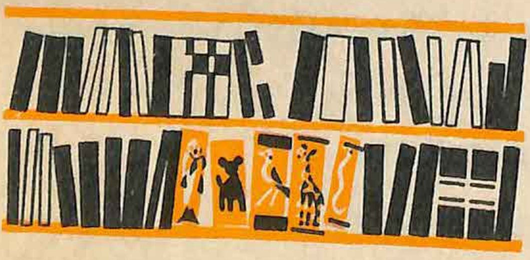
When all the members of one set are also contained in a second set, we say that the members of the smaller set form a *sub-set* of the other.

Let us look at a few drawings.

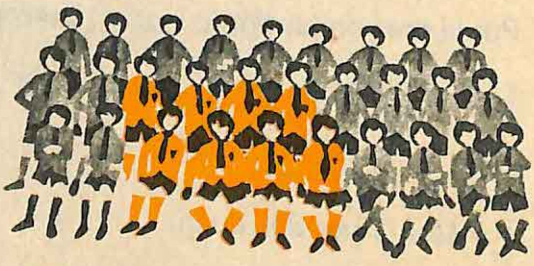
This drawing says that the set of children in your section is a sub-set of the set of children in the class.

Why ?

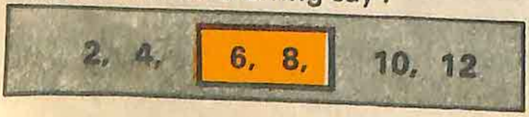
Because all the children in your section are also members of the class.



What does this drawing say ?



What does this drawing say ?



What does this drawing say ?

Make up some sets and sub-sets of your own, and describe them like this:

Set A = {■, ▮}   Set B = {the set of four-sided figures}

Set A is a sub-set of Set B.

Set X = {3, 7, 8}   Set Y = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Set X is a sub-set of Set Y.

More 'shorthand'

As is usual in mathematics we try to find a short way of saying that one set is a sub-set of another. We do this by choosing a sign or symbol. The sign that has been chosen is ⊂ , and means 'is a sub-set of'.

Practise using the new sign.

Set A = {2, 7}   is a sub-set of Set B   {1, 2, 3, 5, 7}

We now write: Set A ⊂ Set B.

Sometimes to make it shorter still we leave out the word Set, and instead of writing Set A ⊂ Set B we write A ⊂ B.

Now make a collection of sets and sub-sets.

Use letters, figures, names, pictures, etc., and write them down in our new short way (Set A ⊂ Set B).

Look at these three sets.

Set X = {a, b, c, d, e, f}   Set Y = {b, c, d}   Set Z = {f}

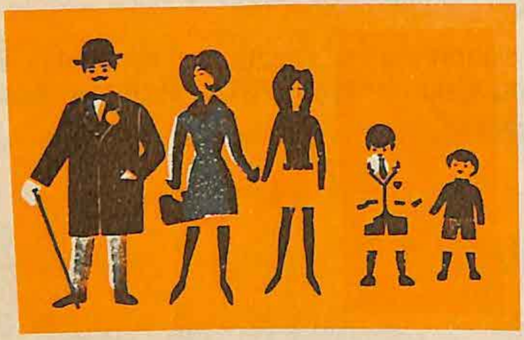
And complete Set Y..........Set X   Set Z..........Set X
or   Y..........X   Z..........X

You see one set can contain *more* than one sub-set.

Let us work at a few more drawings:

This drawing shows the set of the Smith family: father, mother and three children.

This drawing shows the set of the Smith family and its sub-set the Smith children.



This drawing shows the set of the Smith family and its sub-set the Smith boys.

Keeping to the same set but using a different drawing.

This drawing shows the set of the Smith family and the sub-set of the Smith parents.

This drawing shows the set of the Smith family and the sub-set of the Smith girl.

Now make a few drawings of your own showing sets and sub-sets. Here are some ideas for you: (1) Show in a drawing the set of the members of your own family. Make some more drawings now to show some of the sub-sets in your family. (2) Make drawings to show some of the sub-sets of the books in your desk. (3) Make drawings to show the following sets and sub-sets:

{A, G}⊂{A, C, G, H, E}

{▲, ▮}⊂ ■, ◢, ▲, ●, ▮}

{Jan, Feb.}⊂{Jan, Feb, Mar, April}

{white, black}⊂{ red, blue, white, grey, black}

Exercise 14

Name	Age	Colour of hair	Height
Tom	10	fair	150 cm
John	10	red	140 cm
Bill	11	dark	140 cm
Fred	9	dark	150 cm
Joe	11	fair	135 cm
Bert	9	dark	135 cm

This table gives you some information about a set of six boys.

Write the members of the following sub-sets:

1 The sub-set of boys with dark hair.
2 The sub-set of boys who are 10 years old.
3 The sub-set of boys who are 150 cm tall.
4 The sub-set of boys who are fair-haired.
5 The sub-set of fair-haired boys who are 150 cm tall.
6 The sub-set of fair-haired boys who are aged 11.
7 The sub-set of boys who are 11 years old and dark-haired.
8 The sub-set of 9 year old boys who are 135 cm tall.
9 The sub-set of 11 year old boys who are red-haired.
10 The sub-set of boys who are 135 cm tall and dark-haired.

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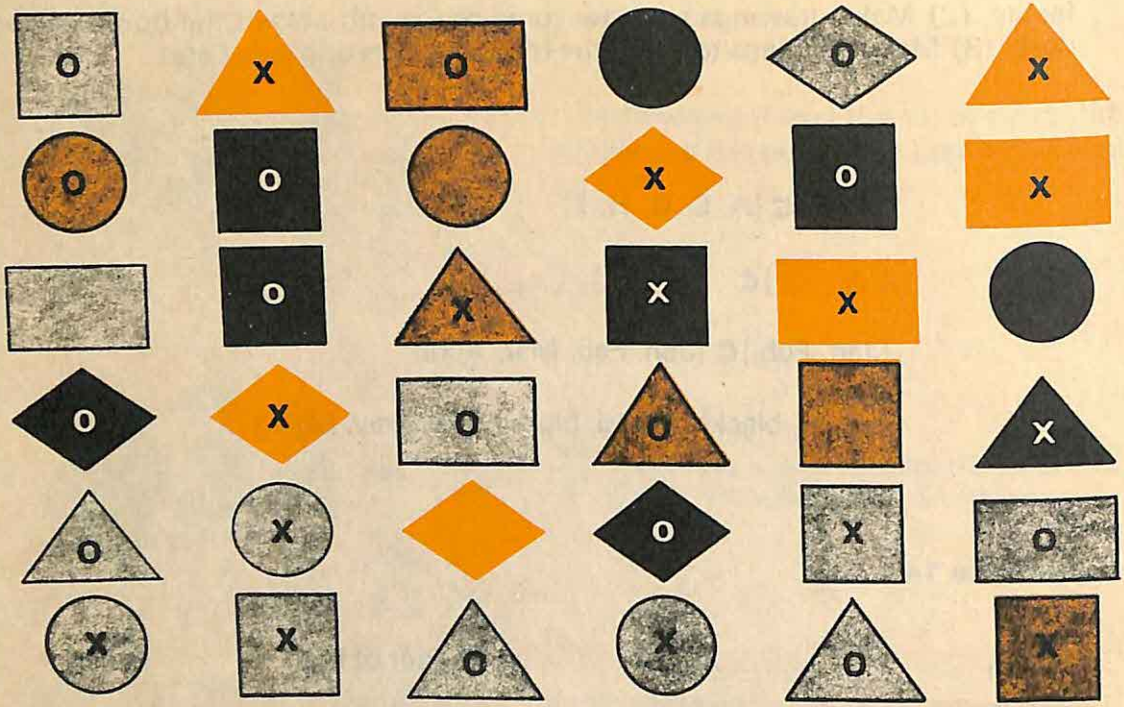

In this chart we have a collection of shapes; some are squares, some rectangles, triangles, circles and diamond shapes.
They are coloured in grey, brown, orange or black.
Some of them have been marked with either a nought (0) or a cross (X). A few are not marked.
You may find it useful to make yourself a set of these figures out of coloured paper or cardboard. Otherwise you can use the chart.

Exercise 15

A. Collect the set of square shapes. Call it Set S.

1 How many members has it?
2 How many members are there in the sub-set of grey squares?
3 Make a drawing of the sub-set of brown squares.
4 How many members has it?
5 Draw the sub-set of orange squares.

B. Collect the set of brown shapes.

1 Draw the sub-set of brown shapes marked with X.
2 How many members in the sub-set of brown shapes are circular?

C. How many members are there in the following sub-sets?

1 The sub-set of grey shapes that are square and marked with 0.
2 The sub-set of grey shapes that are unmarked.
3 The sub-set of grey shapes that are circular and marked 0.
4 The sub-set of grey diamond shapes that are unmarked.

D. Collect the set of unmarked shapes.

1 How many of these would be also in the set of four-sided shapes?
2 List the members of four-sided shapes that are black and unmarked.
3 How many members in the sub-set of unmarked circular shapes are grey?
4 How many of the unmarked shapes coloured orange are also triangular?

How many sub-sets?

One more thing about sub-sets before we leave them for a time.

If a set is a small one it will, of course, not have many sub-sets. If it is a big set it is likely to have many sub-sets.

Sometimes we need to know how many sub-sets can be formed from a given set.

Let us suppose we have a set of three boys: {Bill, Joe, Fred}. How many sub-sets can you make from this set? Write them down.

If you have found all the sub-sets the answer is 8. This may surprise many of you.

Here they are:

A = {Bill, Joe, Fred}

B = {Bill, Joe }

C = {Bill, Fred }

D = {Joe, Fred}

E = {Bill }

F = {Joe}

G = {Fred}

H = { }

The two sub-sets A and H will come as a surprise to you.

The first sub-set A is the same as the set itself. But it is true, isn't it, that all the members of this sub-set A are to be found in the set we started with?

That being so, we may say that any set is a sub-set of itself.

You may find it difficult to understand how the empty set H is a sub-set.

Let us look at the whole thing this way.

We will take the original set of {Bill, Joe, Fred} and ask ourselves in how many ways we could ask them to come to the pictures with us. We could ask *all three;* we could ask *any two of them,* or *one of them,* or we could ask *none of them.*

Do you understand now why {Bill, Joe, Fred} and { } are sub-sets of the set {Bill, Joe, Fred}?

When we ask how many sub-sets can be formed we include the empty set and the original set.

Exercise 16

Find out how many sub-sets in each of the following sets:

1 { }

2 {Susan}

3 {7, 11}

4 {a, b, c}

5 {Ford, Austin, Bentley, Rover}

6 {1, 2, 3, 4, 5}

When you have finished fill in the following table:

Set	No. of members in set	No. of sub-sets
7 { }	0	....
8 {Susan}	1	....
9 {7, 11}	2	....
10 {a, b, c}	3	....
11 {Ford, Austin, Bentley, Rover}	4	....
12 {1, 2, 3, 4, 5}	5	....

Work out how many sub-sets there will be in a set of 6 members.

What does sign $\subset$ mean (e.g. $A \subset B$)?

What do you think this sign means? $\not\subset$

Yes, it means 'is not a sub-set of'.

$A = \{2, 4, 6\}$ $B = \{1, 3, 7, 9\}$

Is A a sub-set of B?

$A \not\subset B$

The Union of Sets

Two brothers went car-spotting for ten minutes.

This was the list made by the elder brother, Tom :

A

Cortina,Volkswagen, Austin Mini, Vauxhall and Morris Oxford.

The younger brother, George, had this list :

B

Cambridge, Zephyr, Rover, Consul and Jaguar.

They made a joint list of all the cars they had seen, like this :

C

Cortina, Volkswagon, Austin Mini, Vauxhall, Morris Oxford, Cambridge, Zephyr, Rover, Consul and Jaguar.

What they did, of course, was to make a *joining or union* of their two sets to make a third set.

Set C is the result of joining, or making a union of Sets A and B.

Look at these lines of sets :

$A = \{3, 4\}$ $B = \{5\}$ $C = \{3, 4, 5\}$

$A = \{c\}$ $B = \{a, b, d\}$ $C = \{a, b, c, d\}$

$A = \{\text{Tom, Dick}\}$ $B = \{\text{John, Joe}\}$ $C = \{\text{Tom, Dick, John, Joe}\}$

$A = \{\text{blue}\}$ $B = \{\text{red, white}\}$ $C = \{\text{blue, red, white}\}$

What have you noticed ?

Yes ; in each line Set C is the same as Set A and Set B joined together.

If all the members of one set are joined to the members of another set to make a new set, the new set is called the *union* of the two sets.

For each line above we could write :

Set C is the Union of Set A and Set B.

Unite these two sets to make a third set :

$\{A, B\}$ and $\{X, Y\}$

$\{8, 2, 3\}$ and $\{5, 1, 9, 6\}$

{ ●, ▲, ■ } and { ▮, ◣, ▬ }

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Oral Exercise

If I say that Set X is the union of Set Y and Set Z, complete the following:

Set X	Set Y	Set Z
4 5 6 7		6 5
	d e f	g h
1, 2, 3, 4, 5, 6	2, 3, 4	
Tom, Joe, Mary, Wilf	Tom	

This is an important thing to know about writing sets:

If we find one or more members are the same in the sets we are going to join together, then we only write down that member (or members) *once* in the union set.

For example:

{Mr. Smith, Joe Smith}

Set A

{Mr. Smith, Mary Smith}

Set B

{Mr. Smith, Joe Smith,
Mary Smith}

Set C

In uniting these Sets A and B, Mr. Smith is in both, but he is the same Mr. Smith and we do not need to write his name down twice in the union set.

THE SYMBOL

Once again we look for a shorthand way of writing all this down. We choose a sign or a symbol to mean 'is the union of'. The sign or symbol that has been chosen is **U**.

So instead of saying Set C is the union of Set A and Set B, we write:

Set C **U** Set A and Set B or better still: C = A **U** B

We have learned quite a few symbols (signs) now. Let us see if you have forgotten any of them.

What do these signs stand for:

∈ means……………………

= means……………………

{ } means……………………

ϕ means……………………

≠ means……………………

⊂ means……………………

U means……………………

∉ means……………………

Here are some examples of the union of sets:

Sets	*Union*
{A, B, C} and {A, D, E}	{A, B, C, D, E}
{5, 6, 7} and {7, 8, 9}	{5, 6, 7, 8, 9}
{●, ▲} and {▮, ●} and {▮, ■}	{■, ▲, ●, ▮}

Make up some sets of your own that have some members in common. Then write and draw the union of them.

Do remember: **If the same thing is a member of two or more sets we record it only once in the union set.**

Exercise 17

Write the union of these sets in the same way as the first which has been done for you:

1 A {11, 12, 13, 14, 15} B {12, 14, 16, 18} A **U** B = {11, 12, 13, 14, 15, 16, 18}

2 C {5, 10, 25, 20, 15} D {30, 35, 40, 45}

3 E {b, r, e, a, d} F {j, a, m}

4 G {2, 4, 6, 8, 10} H {5, 6, 7, 8, 9}

5 I {The odd numbers from 1 to 10}   J {The even numbers from 1 to 10}
6 L {1, 5, 9, 13, 17}   M {3, 6, 9, 12}
7 N {2, 4, 6, 8}   O {1, 2, 3, 4, 5}
8 P {A, B, C, D, E, F}   Q {B, E, G, H}
9 R {B, C, D, E}   S {B, C, D, E}
10 T {A, B, C, D}   U {B, D, C, A}

In the following table there are some blanks.
Can you supply a set that you think could be the missing set ?

Exercise 18

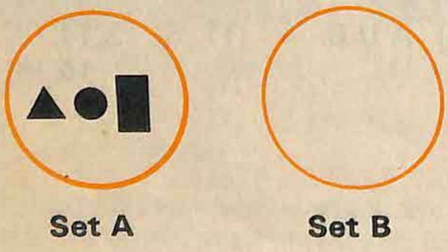
	Set A	Set B	A B (The union)
1		{9, 12, 15, 17}	{8, 9, 10, 11, 12, 15, 17}
2		{4, 5, 6}	{1, 2, 3, 4, 5, 6, 7}
3	{B, D, E}		{A, B, C, D, E, F}
4	{B, I, G}		{L, B, I, G, M}
5	{9, 18, 27, 36}	{Has five members}	{3, 6, 9, 12, 15, 18, 27, 36}

Did you notice something different about Question 9 in Exercise 17 ?

R {B, C, D, E}   S {B, C, D, E}   R ∪ S = {B, C, D, E}

If we join a set with an identical set we get the original set.

Can we join a set to an empty set ?



Set B is an empty set (ø).

If we write down all the members that belong to Set A and Set B

we get ▲ ● ■

This is the same as Set A.

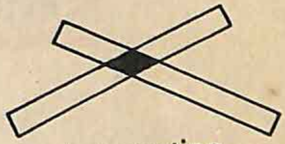
So we say that Set A $\cup \varnothing$ = Set A *or* Set A $\cup \{\ \}$ = Set A

The union of an empty set with another set is the other set itself.

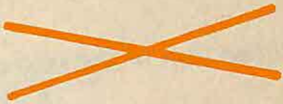
INTERSECTION OF SETS

Find from your dictionary the meaning of the word 'intersection'. You will probably find that it is described as the place where one thing crosses another.

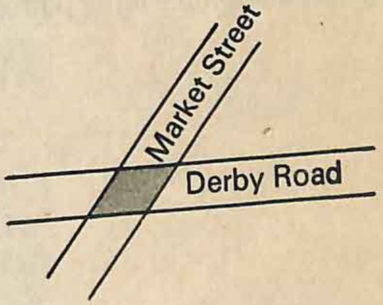
Take two rulers and place one across the other. Make a rough plan of them and shade in the intersection, the place where they cross each other.


Intersection

Here are two straight lines intersecting:



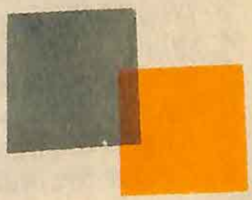
You draw two straight lines intersecting at a different point. Show with an arrow the point of intersection.



The picture on the right shows the intersection of Market Street and Derby Road. You would probably call it the crossroads. American children would call it a street intersection.

Is any part of your own road intersected by another road or street? If so, make a rough sketch showing the roads and mark in the intersection.

Here are two squares intersecting, with the intersection marked.



Draw two intersecting circles and mark in the intersection.

Draw two rough (irregular) shaped figures which intersect. Mark the intersection.

Do the same with two rectangles.

Now let us examine these intersections carefully. Have you noticed that in every case the intersection belongs to both of the things that intersect? Look at the road intersection. Doesn't that piece belong to both roads? Look at the circle intersection. It belongs to both circles doesn't it?

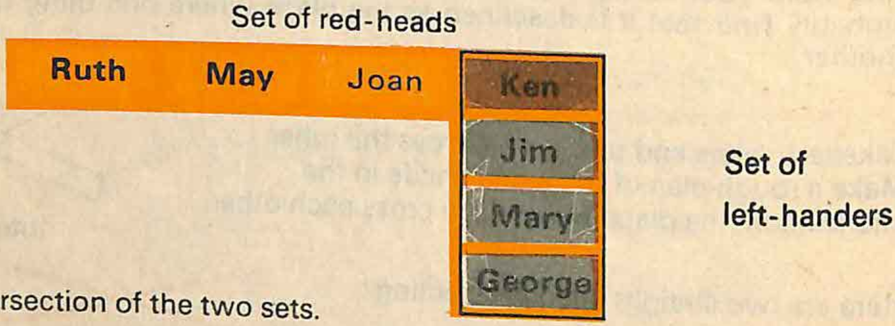
Now sets can intersect in exactly the same way.

That is, part of one set can also be part of another set.

In one class there was a set of four red-headed children:
Ruth, May, Joan and Ken.

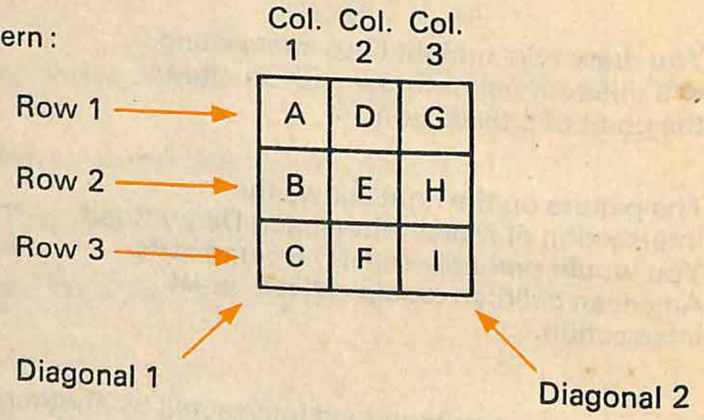
In the same class there were four left-handed children:
George, Jim, Ken and Mary.

You notice that Ken belongs to both sets.



Ken is the intersection of the two sets.

Look at this simple letter pattern:



A	D	G
B	E	H
C	F	I

Now see if you can answer these questions:

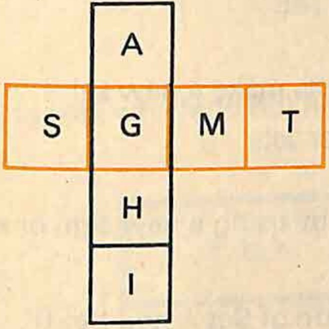
1 What is the intersection of Col. 1 with Row 1?
2 What is the intersection of Row 3 with Col. 2?
3 What is the intersection of Diagonal 1 with Diagonal 2?
4 What is the intersection of Row 2 with Col. 1?
5 What is the intersection of Row 2 with Col. 3?
6 What is the intersection of Row 3 with Diagonal 2?
7 What is the intersection of Col. 1 with Diagonal 1?
8 What is the intersection of Row 1 with Col. 3?

I am sure you will understand now how sets can intersect.

If part of one set is also part of another set then this 'part' is the intersection of the two sets.

Look at these two sets : {A, G, H, I} {S, G, M, T}

The member G is a member of both sets.



This is another view of the same sets. You notice that G is the intersection of the two sets and belongs to both.

We write : Set G is the intersection of Set {A, G, H, I }
and Set {S, G, M, T}.

Look at these diagrams :

Set A	Set B	Set C
5, 10, 15, 20, 25	3, 6, 9, 12, 15, 18	
C, D, E, G, H	A, B, F, H, I	
Bill, Tom, Mary, Sue, Joan	Fred, Tom, Jim, Sue, Michael	

Complete each one so that Set C is the intersection of Set A and Set B.

Now complete these:

Set A	**Set B**	**Intersection Set**
{6, 2, 5, 4}	{6, 3, 7, 4}	{ }
{a, b, c, d, e, f}	{d, e, f, g, h, i}	{ }
{▲, ▬, ●, ▮, ◆}	{▲, ▰, ●, ⬢}	{ }

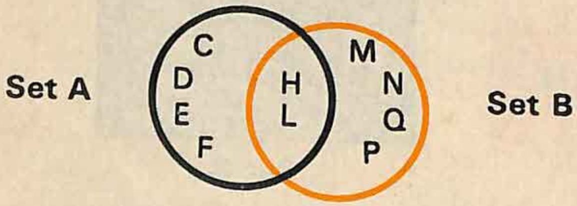
The members that are shared by two or more sets make a new set.
We call the new set the intersection of the other sets.

And again we find a short way of writing this, by using a new sign or symbol.
The sign chosen for 'is the intersection of' is $\cap$.
Thus, instead of writing 'Set C is the intersection of Set A and Set B',
we write: Set C = Set A $\cap$ Set B
or C = A $\cap$ B

{6, 7}	{4, 5, 6, 7}	{6, 7, 8, 9}
Set X	**Set Y**	**Set X**

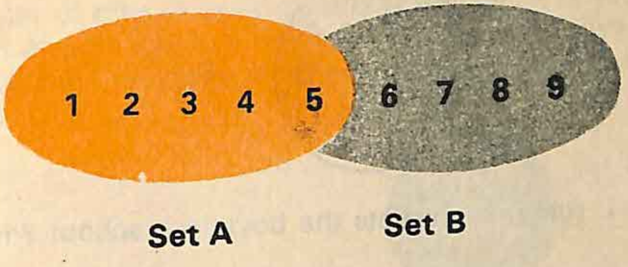
Set X = Set Y $\cap$ Set Z

These intersections are often shown in drawings or diagrams.

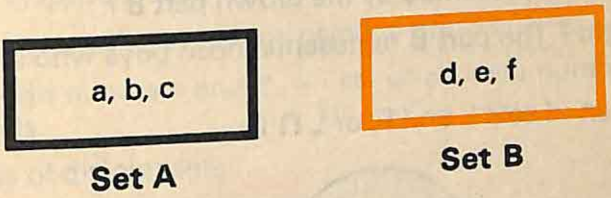


What are the members of circle A (or Set A)?
What are the members of circle B (or Set B)?
Which members are found in both circles?
What is the intersection of Set A and Set B?
Finish this: {H, L} =

Write a statement about this diagram :



What about sets which do not have any members in common ?



What is the intersection of these two sets ?

They have no members in common.

So the intersection of these has no members, and is, therefore, an empty set.

We write : Set A ∩ Set B = ø

If two sets do not share any members, their intersection is an empty set.

Exercise 19

Find the intersecting set of the given sets.

Write your answer properly : A ∩ B =

1 A = {a, e, i, o, u}   B = {a, b, c, d, e}   A ∩ B =

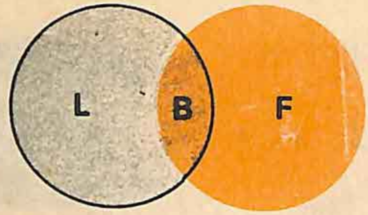
2 C = {even nos. to 10}   D = {odd nos. to 10}

3 E = {c, a, t, s }   F = {e, a, t, s}

4 G = {c, d, 3, 6}   H = { ▲, a, e}   L = {6, f}   M = { }

Find : (a) G ∩ H (b) G ∩ L (c) G ∩ M (d) H ∩ L (e) L ∩ M (f) H ∩ M

Now let us look at a few diagrams showing intersecting sets:



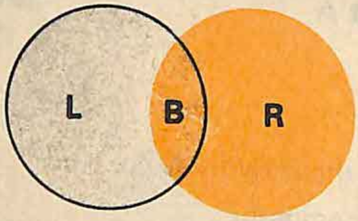
The first shape L (grey) represents the boys in a school who were studying Latin.

The second shape F (orange) is meant to show the boys who were studying French.

What do you think is represented by the brown part B?

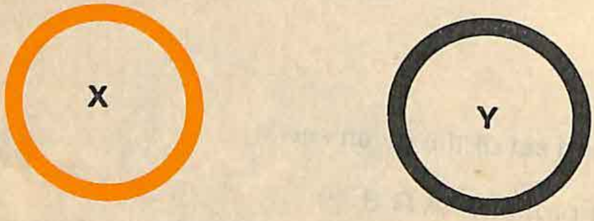
Did you guess right? The part B represents those boys who study both French and Latin.

It is the intersection of sets L and F, or L ∩ F.



In the above diagram L is the set of left-handed children in a class and R is the set of red-headed children.

Describe the set B.



X is the set of light-haired children in a class.

Y is the set of dark-haired children in a class.

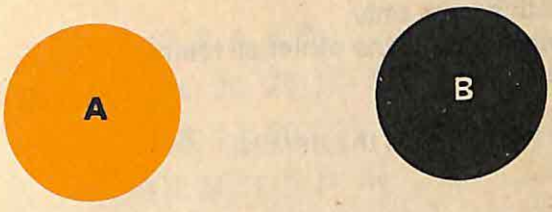
These two sets do not intersect because there are no members of the light-haired group in the dark-haired group.

We say $X \cap Y = \{\ \}$

or $X \cap Y = \varnothing$

Let us have a further look at sets which do not intersect at all.

Such sets are the set of girls in your class and the set of boys. There are no members of the set of girls who are members of the set of boys! The sets are completely separate and independent. When we show them in diagrams they look like this:



We say that Set A is *disjoint* from Set B.

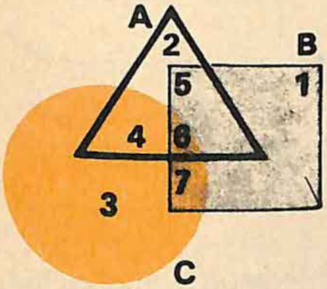
Disjoint means 'not joined', an obvious name to choose.

If X = set of all odd numbers and Y = set of all even numbers then we say they are disjoint sets.

Find some pictures of disjoint sets.

Label them and put them in the Class Book.

Let us study the diagram below. We can learn a lot about intersections from it.



Here in this diagram are three sets intersecting.

List the members of the circle C = { }
i.e. all the figures within the circle.

List the members of the square B = { }

List the members of the triangle A = { }

Now answer these questions:

Exercise 20

1 Which elements are found in both triangle A and circle C?

2 Complete: A ∩ C = { }

3 Which elements are found in both triangle A and square B?

4 Complete: A ∩ B = { }

5 Which elements are common to circle C and square B?

6 Complete: B ∩ C = { }

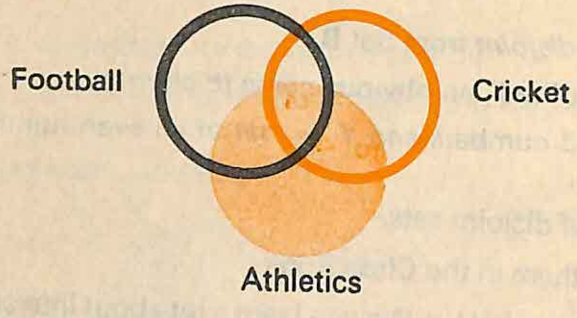
7 Is any element common to all three sets?

8 What do you note specially about elements 1, 2 and 3?

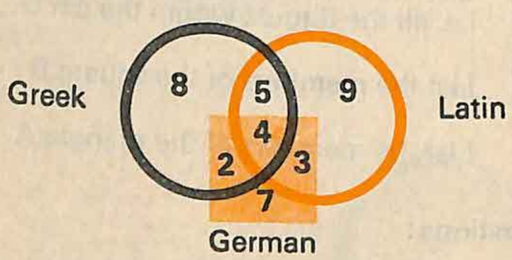
In the top form of a boys' school this is how the boys are distributed in the school teams:

5 boys are in the football team only.
6 boys are in the cricket team only.
3 boys are in the cricket and football teams.
8 boys are in the athletics team only.
1 boy is in the football team and the athletics team.
2 boys are in all 3 teams.

Here is a diagram for you to fill in the details.



How many boys in the cricket team?
How many boys in the football team?
How many boys in the athletics team?
How many boys are in both the cricket and athletics team?



This diagram shows the children taking language courses in a school.

Answer orally the following questions from the diagram:

1 How many children studied both Greek and Latin?
2 How many children studied German only?
3 How many children studied Greek and German?
4 How many children studied Latin and German?
5 How many children in all studied Latin?
6 How many children studied all three languages?
7 How many children studied Greek only?
8 How many children in all studied Greek?
9 How many children studied at least two languages?
10 How many children studied Latin only?

Exercise 21

Complete the third column:

	Set A	Set B	List of members found in both sets
1	{1, 3, 5, 7, 9}	{2, 3, 4, 5, 6}	....
2	{10, 20, 30, 40, 50}	{15, 20, 25, 30, 35}	....
3	{18, 19, 20, 21}	{18, 19, 20, 21}	....
4	{R, E, D}	{N, U, R, E, D, H}	....
5	{a, b, c, d, e, f, g, h}	{b, r, e, a, d}	....
6	{2, 4, 6, 8}	{7, 9, 11, 13}	....
7	{B, A, C, K}	{C, A, B, K}	....
8	{10, 15, 20, 25, 30}	{35, 40, 45, 50}	....

Can you give me some *possible* answers for the missing sets under the heading Set A?

	Set A	Set B	Intersection
9	....	6, 5, 2, 8	6, 2
10	....	o, p, g, r, s, t, b	o, g, b
11	....	1, 3, 9, 27, 81	ø
12	....	1, 2, 3, 4, 5	1, 2, 4

Let us look at number 9. Notice that any set at all that had the elements 6 and 2 would be a possible answer. There is no *one* answer to this question.

Revision

A = {4, 8, 12}

B = {a, b, d}

C = {3}

D = {x, y}

E = {b, d, 4}

F = {3, 4, 8, 12}

Are the following statements true or false?

1 $B \subset E$

2 B and E are disjoint sets

3 C and A are disjoint sets

4 $F = A \cup B$

5 $A \subset F$

6 $\{b, d\} = B \cap E$

7 $C \subset F$

8 A and E are disjoint sets

9 $\{4, 8\} = A \cap F$

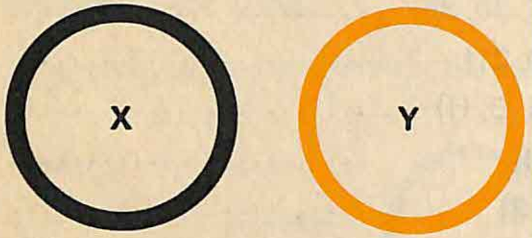
10 $\{a, b, d, b, d, 4\} = B \cup E$

Drawings and Diagrams

(a) What can you say about the sets represented by the diagrams below?

(b) Make up some sets to fit them.

1



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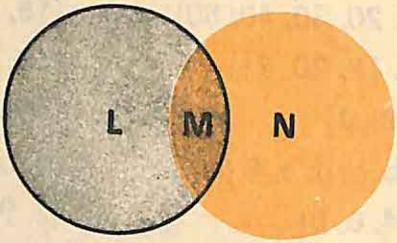

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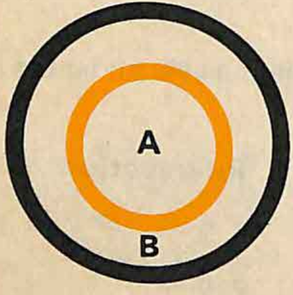

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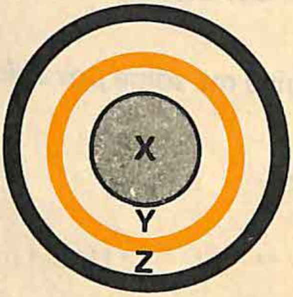


COMMON FACTORS

What is the intersection of these two sets?

A = {9, 10, 11, 12, 13}

B = {8, 10, 12, 14}

If you have remembered rightly your answer is 10, 12, or more correctly,

$A \cap B = \{10, 12\}$

10 and 12 are the members that Set A and Set B have in common.

Let us consider two numbers 18 and 24 and their factors.
Which pairs of factors can you find for the number 18?
I am sure you will find these pretty quickly: 3 × 6, 2 × 9 and 1 × 18.
In short, the set of factors (X) for 18 = {1, 2, 3, 6, 9, 18}

How many pairs of factors for 24?

They are 4×6, 2×12, 3×8, 1×24.

The set of factors (Y) for $24 = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$X \cap Y = 1, 2, 3, 6$

$X \cap Y$ is the set of *common* factors of 18 and 24.

In this exercise follow the same procedure.
Find all the pairs of factors for each number.
Then list the set of factors for each number.
Find the intersection, the set of common factors.

Exercise 22

Find the common factors of:

1 16 and 36
2 14 and 42
3 20 and 30
4 32 and 42
5 27 and 48
6 36 and 64
7 12 and 54
8 8 and 40

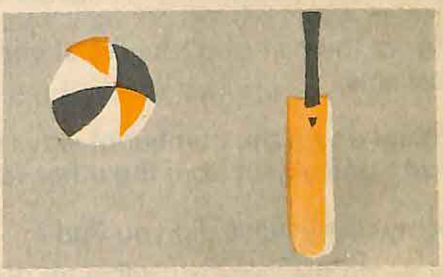
Now look over your answers and say (in each case) which of the common factors was the *greatest* common factor.

Equivalent Sets

MATCHING SETS - ONE-TO-ONE CORRESPONDENCE

Look at these two sets:

Set X



Set Y

In Set X we have Mary and Joe Smith.
In Set Y we have a bat and a ball.

If we say that Mary can play with just one toy from Set Y, and Joe, too, can play with just one toy from Set Y, how can we match them up?

Well, we can match them in this way:

and

giving Joe the ball and Mary the bat. *or*

and

giving Joe the bat and Mary the ball.

We can write this in words.

Joe ⟷ ball

Joe ⟷ bat

Mary ⟷ bat

Mary ⟷ ball

The members of the two sets are paired together one by one.

In olden days shepherds used to count their flocks by matching each sheep with a pebble. This matching gave a one-to-one relationship.

Now let us have three children in front of the class and three different objects (as pen, ruler, book) on the teacher's desk.

Write the names of the set of children on the board and the names of the set of objects.

Write down the number of ways in which each child from the set can be paired with one object from the other set.

How many ways did you find?

Now do the same quickly with these two sets:

A, B, C

7, 8, 9

Start . . .

A ⟷ 7
A ⟷ 8
A ⟷ 9
B ⟷ 7

. . . and go on

If the members of two sets are matched together one with one, we say the sets are in one-to-one correspondence.

We use one-to-one correspondence when we count things. If we say there are three members in a set we mean that the members of the set can be paired together with the set of numbers 1, 2, 3.

Whenever one set has a one-to-one correspondence with another set, the sets are said to be equivalent.

Look at these two sets:

A { Mary, Joe ,Tom }　　B { doll, ball, bat }

These sets *are not equal.* To be equal, you will remember, they have to have the *same* members.

They are equivalent because we can match their members to get one-to-one correspondence like this:

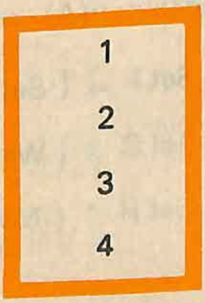
Mary ⟷ doll　　Joe ⟷ ball　　Tom ⟷ bat

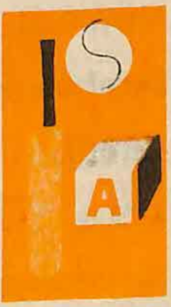
We can say then Set A is equivalent to Set B.

Do keep in mind that being equal and being equivalent are not the same.

Make up three samples of sets that are equivalent (but not equal).

Look at these sets:





Are they equivalent?

Why?

There is no one-to-one correspondence.

They are not equivalent and we call them *non-equivalent* sets.

Make up four examples of *non-equivalent* sets.

Remember again that *identical* or *equal* sets have the *same members* whilst equivalent sets have the *same number of members.*

Name two sets of children in your class that are *identical.*

Name two sets of children in your class that are *equivalent* but not identical.

Give me two sets of children from your class that are *non-equivalent.*

Collect some pictures of equivalent sets and paste them in your class book, and a few non-equivalent sets.

The easiest way to find whether sets are equivalent or not is to count the members of each set.

The number of members in a set is usually called *the number of the set.*

For Set X we would say the number was n(X) (i.e. the number of members in X).

Set X = { 9, 11, 15, 17 }

n(X) = 4

(Number of members in X is 4.)

Exercise 23

Find the number of members in each of these sets:

1 Set A = { Mary, Dora, bat, flower } Like this: n(A) = 4

2 Set B = { Set of months in a year }

3 Set C = { Set of days in a week }

4 Set D = { ½, 3, b, □ }

5 Set E = { Set of toes on a foot }

6 Set F = { Set of letters in alphabet }

7 Set G = { Wickets in a set }

8 Set H = { Members of a soccer team }

9 Set I = { Members of a rugby team }

10 Set K = { Tails on a cow }

11 Set L = { Schools you go to }

12 Set M = { Months with forty days }

13 Set N = { Cows that can whistle }

14 Set O = { Triangles with more than three sides }

More 'Shorthand'

We have, as you might expect, a sign to mean 'is equivalent to'.

It is the sign $\sim$.

A = {1, 2, 3, 4}   B = {2, 4, 6, 8}

A is equivalent to B.

$A \sim B$

How many Sub-Sets?

Turn back to Page 24.

In the exercise at the top of the page we were counting the sub-sets we could get from sets with no members, sets with one member, two members, three members, and so on.

Let us make a table of these sub-sets.

Set	No. of members	Sub-sets with (No. of members)							Total Sub-sets
		0	1	2	3	4	5	6	
{ }	0	1	—	—	—	—	—	—	1
{ Susan }	1	1	1	—	—	—	—	—	2
{ 7, 11 }	2	1	2	1	—	—	—	—	4
{ a, b, c }	3	1	3	3	1	—	—	—	8
{Ford, Rover, Austin, Bentley}	4	1	4	6	4	1	—	—	16
{1, 2, 3, 4, 5}	5	1	5	10	10	5	1	—	32
{d, e, f, g, h, c}	6	1	6	15	20	15	6	1	64

Read from this table the number of sub-sets for sets with :

(1) 3 members.
(2) 4 members.
(3) 5 members.

How many sub-sets do you think there will be for a set with 7 members ?

Look back to Page 23.

There we took the set {a, b, c} and found we could make 8 sub-sets from it.

There was 1 empty set, 3 sets with one member, 3 sets with two members, and 1 set with 3 members (8 in all).

You will see that these figures agree with our table. Can you make a rule about the number of sub-sets that can be formed from a set ?

Finite and Infinite Sets

Some thoughts about numbers :

Is it possible to count all the whole numbers ?

If you write down the biggest number you know, you can always form a bigger one by adding one to make the next number. And the next one. And so it goes on and on without end.

When you get a set of this kind whose members cannot be counted because they go on and on without end, we call it an *infinite* set.

Infinite means 'having no end'.

The set of whole numbers is an infinite set.

The set of odd numbers is an infinite set. You keep on adding 2 to get the next odd number, and so on.

The set of even numbers is also infinite. If you put down the biggest even number you know, you get one bigger by adding 2. The set is endless.

There are, of course, many other infinite sets (sets whose members cannot be counted).

Can you find some for yourself ?

Sets whose members *can* be counted are called *finite* sets.

(a) The set of numbers to 100 can be counted and makes a finite set.
(b) The set of letters A to Z makes a finite set.
(c) The set of even numbers from 10 to 80 can be counted.
    This is a finite set.

By the way, the number of members in some finite sets is so large it would take a lot of room, and would be very tiresome, to write all the members down.

The sets (a), (b) and (c) we have just mentioned are rather big for us to write in all the members between the brackets. When we can see quite clearly the pattern of the set we shorten the list with *three dots,* like this:

(a) The set of numbers to 100 = {1, 2, 3, . . ., 99, 100}

(b) The set of letters A to Z = {A, B, C, . . ., Y, Z}

(c) The set of even numbers from 10 to 80 = {10, 12, 14, . . ., 78, 80}

Exercise 24

State whether this set is finite, infinite or empty:

1 The set of even numbers from 6 to 24.
2 The set of all multiples of 8.
3 The set of prime numbers that are exactly divisible by 2.
4 The set of even numbers that are greater than 12 and less than 14.
5 The set of even numbers that are divisible by 4.
6 The set of all odd numbers.
7 The set of odd numbers that are divisible by 2.
8 The grains of sand on all the beaches of the world.
9 The population of the world.
10 The insects in the world.
11 The natural numbers evenly divisible by 3.

12 Describe this set: {1, 2, 3, 4, 5, . . ., 998, 999}

A = {1, 3, 5, 7, . . ., 37}

13 Is Set A infinite or finite?
14 Is 19 a member of it?
15 Is 18 a member of it?

Infinite sets are usually written down like this:

N = {1, 2, 3, 4, 5, . . .}

Look at these numbers:
Natural numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . .
Even numbers: 2, 4, 6, 8, 10, 12, . . .

Which set do you think has the most members, the set of natural numbers or the set of even numbers?

You are very much inclined to say the set of natural numbers, because the set of natural numbers, as shown, contains all these even numbers.

For many years this problem troubled the greatest mathematicians of the world. But it was solved when George Cantor, the German mathematician wrote about infinite collections.

These writings were the beginning of our knowledge about sets.
He showed that for every natural number there was a corresponding even number like this:

Natural Numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, . . . to infinity
↕ ↕ ↕ ↕ ↕ ↕ ↕ ↕ ↕
Even Numbers: 2, 4, 6, 8, 10, 12, 14, 16, 18, . . . to infinity

It is the same with the square numbers:

Natural Numbers: 1, 2, 3, 4, 5, 6, . . . to infinity
↕ ↕ ↕ ↕ ↕ ↕
Square Numbers: 1, 4, 9, 16, 25, 36, . . . to infinity
1^2 2^2 3^2 4^2 5^2 6^2

It makes you think!

Revision

I think it is about time we went back to look over the new things we have learned.

A *set* is a collection of things.

The things that belong to a set are called the *members* of a set.

When we write down sets we put the members in a diagram like a circle or a ring, or we write them in brackets or braces ({S}).

The sign for 'is a member of' is ∈.

The sign for 'is not a member of' is ∉.

We usually use a capital letter to name a set.

Some sets have many members, others a few, some sets have no members at all. We call these empty sets or null sets.

The sign we use for an empty set is { } or ø.

If two or more sets have exactly the same members then we say the sets are *equal or identical.* It doesn't matter about the order of the members of the set as long as they are the same members. If we have two sets that are equal we write $A = B$.

When all the members of one set are also contained in a second set, we say that the members of the smaller set form a *sub-set* of the other.

We have a sign for this, $A \subset B$ means 'A is a sub-set of B'.

$A \not\subset B$ means, of course, that A is *not* a sub-set of B.

When counting the sub-sets that can be formed from any set we include the empty set and the set itself.

We found that from a set which had no members we could make one sub-set, from a set with one member we could make two sub-sets.

A set of two members had 4 sub-sets.
A set of three members had 8 sub-sets.
A set of four members had 16 sub-sets.
A set of five members had 32 sub-sets.

Joining sets together to make a third set is called the *union* of sets. There is a sign for it $\cup$. $A \cup B = C$ means Set C is the joining together of Set A and Set B.

If we find one or more members are the same in the sets we are going to join together, then we only write down that member *once* in the union set.

If we join a set to an empty set the union is *the set itself.*

We found that some sets crossed or intersected each other, where part of one set was also to be found in another. The part that was in both sets was a new set which we called the intersection. The sign we use for intersection is $\cap$. If Set A = Set B $\cap$ Set C, then A is the set of things that B and C have in common.

Sets that do not touch or intersect are called *disjoint sets.*

Common factors of numbers are really the intersection of the factors of each of these numbers.

Equivalent sets must not be confused with equal sets. In *equal sets* there are the *same members* in each set. In *equivalent sets* we have *the same number* of members in each set.

The symbol we use to say 'one set is equivalent to another' is $\sim$. $A \sim B$. (A has the same number of members as B.)

Numbers that go on and on, without end, are called *infinite numbers.*

So too, if we get a set whose members cannot be counted we call it an *infinite set.*

Sets whose numbers can be counted are *finite sets.*

The Universal Set

We have collected quite a variety of sets and sub-sets so far: sets and sub-sets of many kinds of numbers, letters, days, months, names, shapes, etc.

Each of these sets is itself a part of a bigger, greater set.

For example: (1) the set of even numbers from 10 through to 20 is a part of the set of all numbers in the universe. This is what we call the *universal* set of numbers. The universal set is a kind of parent set.

(2) The set of footballers in your class team is part of the set of *all footballers.*

(3) The set of cars that the two brothers spotted (Page 25) is part of the set of all cars.

(4) The set of children in your class is part of the set of all the children in the school.

In this last example it would be equally true to say that the set of children in your class was part of the universal set of *all children.*

Sometimes, then, we have to say exactly what is meant by the universal set.

Suppose, for example, that the children in your section are 11 years of age, clever and British, your set may belong to the universal set of:

(1) all 11-year old children,
(2) all clever children,
(3) all British children,

or

(4) all children.

If the universal set was not described exactly we would take it to mean the universal set of *all children.*

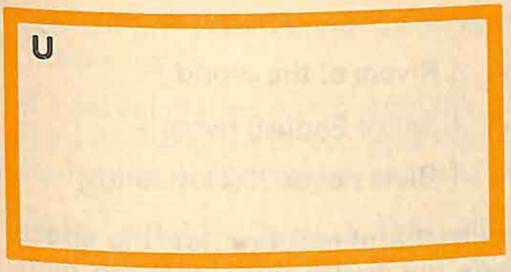
We usually use the letter U as the short for universal set, although you will find that some people use E (or $\mathcal{E}$). E is an abbreviation for 'ensemble' which is the French word for set.

Find a universal set for each of the following sets:

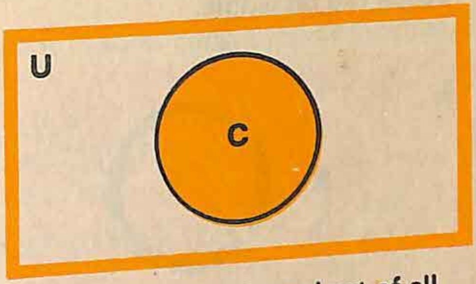
1 The set of odd numbers from 20 to 30.
2 The set of red scarves.
3 The set of children in your handwork/needlework class.
4 The set of vowels.
5 The set of science books in your class library.

Diagrams

In diagrams the universal set is usually shown by a rectangle and its various sub-sets by a circle. All the members of this set are considered as points in or on the rectangle.

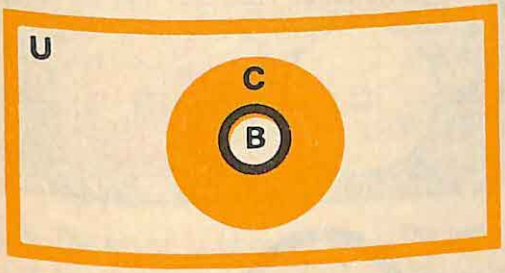


Let this rectangle represent the universal set of all the people in your town.



This shows the universal set of all the people in your town and one of its sub-sets, the children of the town.

The sub-set of children in the town can have its own sub-sets, as, for example, the set of children under school age. This next diagram shows the universal set of all people in the town, sub-set C of all the children in the town, and B the sub-set of children under school age.



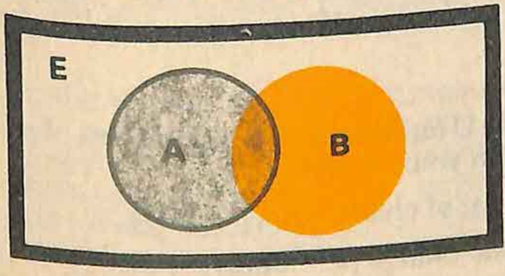
Are you quite sure you can read this diagram? It is saying that the set of children under five (B) is part of the set of children in the town (C) and the set of children (C) is part of the universal set of all the people in the town.

Remembering our signs, we could write:

$$B \subset C \subset U$$

These diagrams we use to illustrate sets are called Venn diagrams. They are so called after the man who first used them, John Venn, a great mathematician who lived about a hundred years ago.

Look at this Venn diagram:



(a) The rectangle E shows the set of all people in your town.

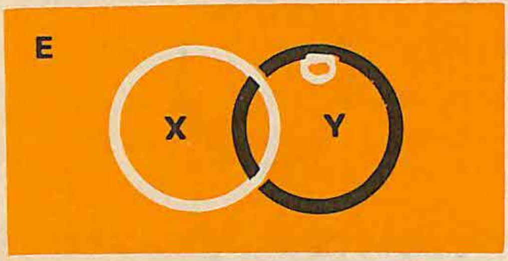
(b) Circle A shows the set of people in your town who are under 25 years of age.

(c) Circle B shows the set of people in the town who are members of the public library.

Can you say:

1 What set of people are shown in the intersection, (the brown part)?

2 Describe the set of people who are shown by the space outside the circles but inside the rectangle.



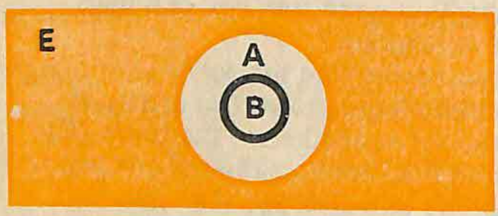
E = { Rivers of the world }
X = { Set of English rivers }
Y = { Rivers over 200 km long }

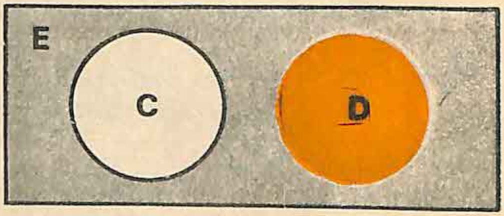
Describe the intersection, and the space outside the circles but inside the rectangle.

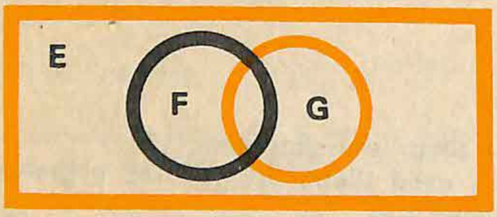
Draw a Venn diagram to illustrate these sets:

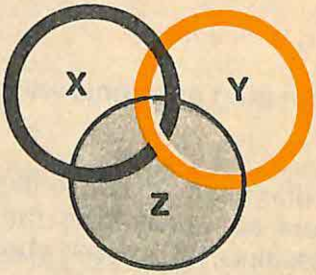
E = {all cars}   F = {Ford cars}   G = {Consuls}

Write a statement about your diagram.
Now here is a chance for you to show your knowledge of set diagrams.
Make up sets which will fit in with each of the following diagrams:





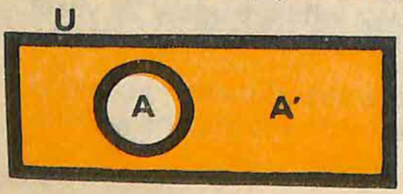




THE COMPLEMENT OF A SET

There is another important set we have not explained yet.

A diagram will help you to understand it.



The rectangle U represents the universal set of all the children in your school.

Set A is the set of children in your class.

What does the orange part represent then?

Yes, it represents all the rest of the children who make up (or complete) your school.

It is the set that completes A to make up the universal set of all the children in the school.

We call it the complement set. A complement is something that 'completes' or 'makes up'.

If a set is labelled A, we label the complement A'.
If a set is labelled D, we label the complement D' and so on.
Remember that A and A' make U (the universal set).

To put it another way: $A \cup A' = U$ Note too that $A \cap A' = \emptyset$

Exercise 25

Complete this table:

Universal Set U	Set X	Set X' (the complement)
1 Set of children in your school	The set of girls	
2 Set of children in your school	The set of first-year children	
3 Set of children in your school	The set of left-handed children	
4 Set of children in your school	The set of children who live in .... Road	
5 The set of months of the year	The set of summer months	
6 The set of whole numbers	The set of odd numbers	
7 The set of letters of the alphabet	The set of vowels	
8 The set of people in the town	The set of children going to school	
9 The set of four-legged animals	The set of horses	

Operations with Sets

In this section we look at some rules that numbers obey. Most of them will be well known to you, although you may not know them as rules. They are things that you have known about numbers for a long time, but never thought about them as 'rules'. Many of you boys have played football for years and know well how it should be played, but have never bothered to read a book of rules. It is often a good thing to know the laws well.

When we have studied some of these rules about numbers we will have a look at sets to see if they obey the laws like numbers do.

You have learned, I think, that we can perform *operations* with sets in much the same way as we do with numbers.

When we add, subtract, multiply and divide numbers we perform *operations* with numbers.

When we work with sets we use operations. The two operations we have learned about are *union and intersection.*

We hear a good deal of talk these days about *binary operations.* Binary means two. A binary operation simply means taking two numbers and operating on them (as adding or multiplying them) to get a third number.

Anyway, let us get back to our rules about numbers.

Rule 1

You will say you have known about the first relation between numbers for years. But let us make a rule about it.

For every pair of numbers it is true to say that we can write:

$$2 + 3 = 3 + 2$$
$$5 + 2 = 2 + 5$$
$$9 + 6 = 6 + 9, \text{etc.}$$

If we are adding a pair of numbers it doesn't matter at all if we change the numbers round: the answer is just the same.

Mathematicians say 'commute the number' instead of 'change the number'.

This rule is called, then, the *commutative law of addition*. If we let the letter a stand for any natural number and the letter b stand for any other natural number, then: $a + b = b + a$.

(Not all binary operations obey this commutative law. Division does not. You know that $8 \div 2$ is not the same as $2 \div 8$.)

Rule 2

This also is a simple rule, but be sure there will be a big word in it!

We talked in Rule 1 about addition performed on a *pair* of numbers. We can, as you know, add together more than two numbers.

If we add three numbers, we usually add together two of them and then to the sum of these two we add the third number.

To take an example: 3 and 4 and 6.

We usually say 3 and 4 are 7, and 6 make 13.

We could write it down this way $(3 + 4) + 6 = 13$. We could have got the same answer by first adding the 4 and 6 and then adding the 3.

We would write it down in this way: $3 + (4 + 6) = 13$.

To put it another way:

$$\begin{array}{r}3\\4\\+\ 6\\\hline\\\hline\end{array}$$

We get the same answer whether we add from top to bottom or from bottom to top.

If we use letters instead of figures, we can say: $(a + b) + c = a + (b + c)$

The rule says, then, that in the first step of the addition we are free to associate (or join) the middle number either with the number on the right or the number on the left.

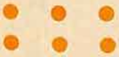
Because we can associate (join) the numbers in any order we call this the *associative law of addition.*

Rule 3

Read through Rule 1 again, the commutative law of addition.

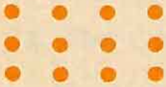
In the last sentence we said the rule didn't work for division.

Does it work for multiplication?



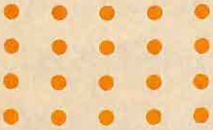
We could write this down as three twos or two threes,

or $3 \times 2 = 2 \times 3$



This is three fours or four threes.

or $3 \times 4 = 4 \times 3$.



Four fives or five fours.

or $4 \times 5 = 5 \times 4$,

and so on.

If we use letters instead of figures we would write:

$$a \times b = b \times a.$$

This is known as the *commutative law of multiplication.*

Rule 4

Read through Rule 2 again (the associative law of addition).

Does it also work for multiplication?

Let us take an example: $2 \times 3 \times 6$.

Does it matter which way we associate the numbers?

Is $(2 \times 3) \times 6$ the same as $2 \times (3 \times 6)$?
$6 \times 6 = 36$ $\quad$ $2 \times 18 = 36$

Try this out with a number of examples.

You will find in each case that it doesn't matter whether we associate the middle number with the number on the left or the right.

This is the *associative law of multiplication.*

Rule 5

x x x x x   x x x x

x x x x x   x x x x

x x x x x   x x x x

This diagram shows 3 times 9 or $3 \times (5 + 4)$

It also shows   x x x x x   x x x x

x x x x x   and   x x x x

x x x x x   x x x x

$(3 \times 5) \quad + \quad (3 \times 4)$

So $3 \times (5 + 4) = (3 \times 5) + (3 \times 4)$

Try this: Does $4 \times (2 + 3) = (4 \times 2) + (4 \times 3)$?

Try more examples.

You will find always that:

$a \times (b + c) = (a \times b) + (a \times c)$

This is called the *distributive law of multiplication.*

It is so called because the multiplier (the number you are multiplying by) can be *distributed* or spread out among each of the items in the number you are multiplying.

e.g. $3(5 + 2 + 6)$ is the same as $(3 \times 5) + (3 \times 2) + (3 \times 6)$

The multiplier (3) is distributed to each of the numbers 5, 2 and 6.

Let us now have a look at the operations we have done so far with sets and see if they obey any of these laws we have learned about numbers.

The best way of going about our task is to take each law, one by one, and see if the rule applies to sets.

1 The commutative law of addition (union)

Union is the addition operation with sets:

Does $X \cup Y = Y \cup X$?

If Set $X = \{2, 4, 5, 8, 9\}$ and $Y = \{2, 3, 5, 9\}$

$X \cup Y = \{2, 3, 4, 5, 8, 9\}$

$Y \cup X = \{2, 3, 4, 5, 8, 9\}$

These are the same sets, so Law 1 has been obeyed for the union of sets.

2 The associative law for union

Is $(X \cup Y) \cup Z$ the same thing as $X \cup (Y \cup Z)$?

Let us suppose Set $X = \{a, b, c\}$, Set $Y = \{c, d\}$, Set $Z = \{a, c\}$.

Then $X \cup Y = \{a, b, c, d\}$ and $(X \cup Y) \cup Z = \{a, b, c, d\}$.

$X = \{a, b, c\}$, $(Y \cup Z) = \{a, c, d\}$, and $X \cup (Y \cup Z) = \{a, b, c, d\}$.

These are the same sets so Law 2 has been obeyed for the union of sets.

3 The commutative law for intersection

Intersection in sets is similar to multiplication in numbers.

We wish to know then if $X \cap Y = Y \cap X$.

Let us use the sets we used to verify Law 1.

$X = \{2, 4, 5, 8, 9\}$ $Y = \{2, 3, 5, 9\}$

$X \cap Y = \{2, 5, 9\}$

$Y \cap X = \{2, 5, 9\}$

These are the same sets so Law 3 has been obeyed for the intersection of sets.

4 The associative law for intersection

Is $(X \cap Y) \cap Z$ equal to $X \cap (Y \cap Z)$?

Let $X = \{a, b, c\}$, $Y = \{c, d\}$, $Z = \{a, c\}$.

$(X \cap Y) \cap Z = \{c\} \cap \{a, c\} = \{c\}$

$X \cap (Y \cap X) = \{a, b, c\} \cap c = \{c\}$

These sets are equal and so Law 4 is obeyed for intersection.

5 The distributive law

For numbers the rule is $a \times (b + c) = (a \times b) + (a \times c)$

(the multiplier is distributed).

For sets, does $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$?

In words: Is the intersection of Set X with the union of Y and Z = intersection of X and Y + intersection of Y and Z?

Let $X = \{a, b, c\}$ $\quad Y = \{c, d\}$ $\quad Z = \{a, c\}$

$X = \{a, b, c\}$, $Y \cup Z = \{a, c, d\}$, their intersection $= \{a, c\}$.

$X \cap Y = \{c\}$, $X \cap Z = \{a, c\}$, their union $= \{a, c\}$.

These two set are the same, so it is true that

$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

The distributive law does apply for sets.

Let us sum up these laws of operations

In the table below you will find the laws described for Arithmetic, Algebra and Sets.

I have put the two commutative laws (add and multiply) together and the two associative laws (add and multiply) together.

1 Commutative law or law of order

The order in which we add two numbers or multiply them makes no difference to the sum or product.

Arithmetic	*Algebra*	*Sets*
$2 + 3 = 3 + 2$	$a + b = b + a$	$A \cup B = B \cup A$
$5 \times 3 = 3 \times 5$	$ab = ba$	$A \cap B = B \cap A$

2 Associative law or law of grouping

The way in which we group together numbers or quantities we are going to add or multiply makes no difference to the sum or product.

Arithmetic	*Algebra*	*Sets*
$(2 + 3) + 4 = 2 + (3 + 4)$	$(a + b) + c = a + (b + c)$	$(A \cup B) \cup C = A \cup (B \cup C)$
$(3 \times 4) \times 5 = 3 \times (4 \times 5)$	$(ab)\,c = a\,(bc)$	$(A \cap B) \cap C = A \cap B(B \cap C)$

3 Distributive law

The number that multiplies a sum multiplies each number in the sum.

Arithmetic	*Algebra*	*Sets*
$3(4 + 5) = (3 \times 4) + (3 \times 5)$	$a(b + c) = ab + ac$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

These laws are very useful in solving problems with computers. When all the information is collected to be fed into the machine, the sets of information are collected in unions and intersections. The unions and intersections show how the circuits in the computer must be arranged to answer the questions we ask of it.

Variables

You will constantly be meeting variables in your work with mathematics.

Your dictionary will tell you that variables are things which vary or change.

Sometimes we know the set that something belongs to without being able to say which particular element or member it is.

If your car breaks down you will want something which is ϵ {tools}.

If you are thirsty you would like something which is ϵ {drinks}.

In these cases we have named the set but not the particular member of the set.

Since we would like to give it a name we usually choose some letter as x, y, or n. The item x, y or n is then a variable, because it can vary. It may be one of several things.

If x, or y, or n belongs to a number set it is a *numerical variable.*

Let us suppose n ϵ {odd numbers}. What value can n have? It could be any one of these: 1, 3, 5, 7, 9, 11, . . .

If n ϵ {factors of 24} what value can n have? It could be 1, 2, 3, 4, 6, 8, 12 or 24.

Find out what value these variables can have:

1 $a \in$ {even numbers less than 10}
2 $b \in$ {multiples of 5 which are less than 16}
3 $c \in$ {odd numbers 10 to 20}
4 $d \in$ {numbers ending in 4}
5 $e \in$ {odd numbers that are multiples of 4}
6 $f \in$ {odd numbers that are multiples of 3 and less than 20}
7 $g \in$ {prime numbers greater than 20 and less than 30}

INEQUALITIES

We often use, as in the above examples, the expressions 'greater than' or 'less than'. We have signs (or symbols) to show that one thing is greater or less than another.

$>$ stands for 'greater than'
$<$ stands for 'less than'

The short forms for 6 is less than 9, and 10 is less than 15 are:

$6 < 9$ $\qquad$ $10 < 15$

So instead of writing 5 is greater than 4, or 7 is greater than 5, we write

$5 > 4$ $\qquad$ $7 > 5$

Exercise 26

Put the correct sign between these pairs:

1 $\quad$ 8 $\quad$ 10
2 $\quad$ 9 $\quad$ 6
3 $\quad$ 5 $\quad$ 4
4 $\quad$ 3 $\quad$ 7
5 $\quad$ 1 $\quad$ 2
6 $\quad$ 81 $\quad$ 85
7 $\quad$ 101 $\quad$ 110
8 $\quad$ $\frac{1}{2}$ $\quad$ $\frac{1}{4}$
9 $\quad$ 1.01 $\quad$ 1.1
10 $\quad$ 5 $\quad$ 5

Exercise 27

What value can the variable have in each of the following pairs, which are not equal, if the variable is a whole number:

1 $\quad$ $a < 3$
2 $\quad$ $y > 21$
3 $\quad$ $b < 5$
4 $\quad$ $n > 7$
5 $\quad$ $g > 9$
6 $\quad$ $x + 1 > 6$
7 $\quad$ $x + 1 < 4$
8 $\quad$ $x + 2 > 5$
9 $\quad$ $x + 4 < 11$
10 $\quad$ $x + 3 < 5$

Answers to Exercises

Exercise 1 Page 4
(1) 4 (2) 2 (3) 2 (4) no

Exercise 2 Page 5
(1) . . . four . . . set
(2) member
(3) a kite, a bat, a ball
(4) member
(5) figures (or odd numbers)
(6) members
(7) member . . . set
(8) 1, 5, 7, 9

Exercise 3 Page 5
Individual answers

Exercise 4 Page 6
(1) 1
(2) $\{\text{February}\}$ 1 member
(3) 1
(4) 1
(5) 1, $\{8\}$, 1
(6) 1, $\{5\}$, 1

Exercise 5 Page 9
(1) $\{w, x, y, z\}$
(2) $\{\text{Tuesday, Thursday}\}$
(3) $\{6, 8, 10, 12, 14, 16\}$
(4) $\{11, 13, 15, 17, 19\}$
(5) $\{2, 4, 6, 8, 10\}$
(6) $\{16\}$
(7) $\{10, 11, 12, 13, 14, 15, 16, 17\}$

(8) $\{51, 52, 53, 54, 55, 56, 57, 58, 59, 60\}$

(9) $\{110, 111, 112, 113, 114, 115\}$

(10) $\{697, 698, 699, 700, 701, 702, 703\}$

(11) $\{a, e, i, o, u\}$ (12) $\{\text{Saturday, Sunday}\}$

(13) individual answers (14) individual answers

(15) $\{1, 2, 3, 4, 6, 12\}$ (16) $\{5, 10, 15, 20, 25, 30, 35\}$

(17) $\{7, 21, 35, 49\}$ (18) $\{1, 2, 4, 5, 10, 20\}$

Exercise 6 Page 10

(1) 26

(2) 5

(3) individual answers

(4) 12

(5) 7

(6) $\{J, F, M, A, S, O, N, D\}$

(7) $\{12, 15, 18\}$

(8) $\{4, 8, 12, 16\}$

Exercise 7 Page 10

(1) (c) : 6

(2) (c) : 2

(3) (b) : 5

(4) (a) : 4

Exercise 8 Page 11

There will be a few acceptable variants to these answers.

(1) D. The set of the last four letters of the alphabet.

(2) E. The set of the odd numbers less than ten.

(3) F. The set of the months whose names begin with J.

(4) G. The set of even numbers greater than 8 and less than 22.

(5) H. The set of counting numbers less than 400 ending in 00.

(6) I. The set of counting numbers less than 26 exactly divisible by 5.

(7) J. The set of counting numbers less than 11.
(8) K. The set of roman numerals greater than IV and less than XI.
(9) L. The set of vowels.
(10) M. A set of four-sided figures.
(11) N. The set of the days of the week whose names start with the letter T.
(12) X. A set of writing materials.
(13) Y. A set of vehicles.
(14) Z. The set of numbers between 10 and 30 exactly divisible by 4.

Exercise 9 Page 13

(1) $\in$
(2) $\in$
(3) $\notin$
(4) $\in$
(5) $\notin$
(6) $\in$
(7) $\notin$
(8) $\in$
(9) $\notin$
(10) $\in$
(11) $\notin$
(12) $\in$
(13) $\notin$
(14) $\notin$
(15) $\in$
(16) $\in$

Exercise 10 Page 14

Answers orally

Exercise 11 Page 16

Answers orally

Exercise 12 Page 16

(1) yes
(2) yes
(3) yes
(a) set M $\neq$ set N
(b) set D $\neq$ set F
(c) set G $\neq$ set H
(d) set J $\neq$ set K
(4) no
(5) yes
(6) no

Exercise 13 Page 17

(1) $\neq$
(2) $=$
(3) $=$
(4) $\neq$

Exercise 14 Page 21

(1) $\{\text{Bill, Fred, Bert}\}$
(2) $\{\text{Tom, John}\}$
(3) $\{\text{Tom, Fred}\}$
(4) $\{\text{Tom, Joe}\}$
(5) $\{\text{Tom}\}$
(6) $\{\text{Joe}\}$
(7) $\{\text{Bill}\}$
(8) $\{\text{Bert}\}$
(9) $\{\ \}$ or $\varnothing$
(10) $\{\text{Fred}\}$

Exercise 15 Page 22

A. (1) 9
(2) 3
(3) 2
(4) 2
(5) 0

B. (1) 2
(2) 2

C. (1) 1
(2) 1
(3) 0
(4) 0

D. (1) 3
(2) 0
(3) 0
(4) 0

Exercise 16 Page 24

(1) 1
(2) 2
(3) 4
(4) 8
(5) 16
(6) 32
(7) 1
(8) 2
(9) 4
(10) 8
(11) 16
(12) 32

Exercise 17 Page 27

(1) $A \cup B = \{11, 12, 13, 14, 15, 16, 18\}$
(2) $C \cup D = \{5, 10, 15, 20, 25, 30, 35, 40, 45\}$

(3) $E \cup F = \{b, r, e, a, d, j, m\}$

(4) $G \cup H = \{2, 4, 5, 6, 7, 8, 9, 10\}$

(5) $I \cup J = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(6) $L \cup M = \{1, 3, 5, 6, 9, 12, 13, 17\}$

(7) $N \cup O = \{1, 2, 3, 4, 5, 6, 8\}$

(8) $P \cup Q = \{A, B, C, D, E, F, G, H\}$

(9) $R \cup S = \{B, C, D, E\}$

(10) $T \cup U = \{A, B, C, D\}$

Exercise 18 Page 28

(1) $\{8, 10, 11\}$ (2) $\{1, 2, 3, 7\}$

(3) $\{A, C, F\}$ (4) $\{L, M\}$

(5) {3, 6, 12, 15 and any *one* of the other numbers}

Exercise 19 Page 33

(1) $A \cap B = \{a, e\}$ (2) $C \cap D = \{\ \}$ or $\emptyset$

(3) $E \cap F = \{a, t, s\}$

(4) (a) $G \cap H = \{\ \}$ (b) $G \cap L = \{6\}$

(c) $G \cap M = \{\ \}$ (d) $H \cap L = \{\ \}$

(e) $L \cap M = \{\ \}$ (f) $H \cap M = \{\ \}$

Exercise 20 Page 35

(1) 4, 6
(2) $A \cap C = \{4, 6\}$
(3) 5, 6
(4) $A \cap B = \{5, 6\}$
(5) 6, 7
(6) $B \cap C = \{6, 7\}$
(7) 6
(8) Do not connect with any other set.

Exercise 21 Page 37

(1) 3, 5
(2) 20, 30
(3) 18, 19, 20, 21
(4) R, E, D
(5) b, e, a, d
(6) —
(7) B, A, C, K
(8) —

Exercise 22 Page 39

(1) 1, 2, 4 G.C.F. = 4
(2) 1, 2, 7, 14 G.C.F. = 14
(3) 1, 2, 5, 10 G.C.F. = 10
(4) 1, 2 G.C.F. = 2
(5) 1, 3 G.C.F. = 3
(6) 1, 2, 4 G.C.F. = 4
(7) 1, 2, 3, 6 G.C.F. = 6
(8) 1, 2, 4, 8 G.C.F. = 8

Exercise 23 Page 42

(1) $n(A) = 4$
(2) $n(B) = 12$
(3) $n(C) = 7$
(4) $n(D) = 4$
(5) $n(E) = 5$
(6) $n(F) = 26$
(7) $n(G) = 2$
(8) $n(H) = 11$
(9) $n(I) = 15$ (or 13)
(10) $n(K) = 1$
(11) $n(L) = 1$
(12) $n(M) = 0$
(13) $n(N) = 0$
(14) $n(O) = 0$

Exercise 24 Page 45

(1) Finite
(2) Infinite
(3) Finite (2 is a prime number)
(4) Empty or ø
(5) Infinite
(6) Infinite
(7) Empty or ø
(8) Finite
(9) Finite
(10) Finite
(11) Infinite
(12) The set of counting numbers 1 to 999
(13) Finite
(14) Yes
(15) No

Exercise 25 Page 51

(1) The set of boys.
(2) The set of children in other years.
(3) The set of right-handed children.
(4) The set of children living in other roads.
(5) The set of months in Spring, Autumn, and Winter.
(6) The set of even numbers.
(7) The set of consonants.
(8) The set of adults and children under school age.
(9) The set of all four-legged animals other than horses.

Exercise 26 Page 58

(1) <
(2) >
(3) >
(4) <
(5) <
(6) <
(7) <
(8) >
(9) <
(10) =

Exercise 27 Page 58

(1) 1, 2
(2) 22, 23, 24, . . . infinity
(3) 1, 2, 3, 4
(4) 8, 9, 10, . . . infinity
(5) 10, 11, 12, . . . infinity
(6) 6, 7, 8, ... infinity
(7) 1, 2
(8) 4, 5, 6,. . . infinity
(9) 1, 2, 3, 4, 5, 6
(10) 3, 4, 5, . . . infinity

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Understanding SETS

Understanding NUMBER

Understanding SHAPES & SOLIDS

Understanding GRAPHS & STATISTICS

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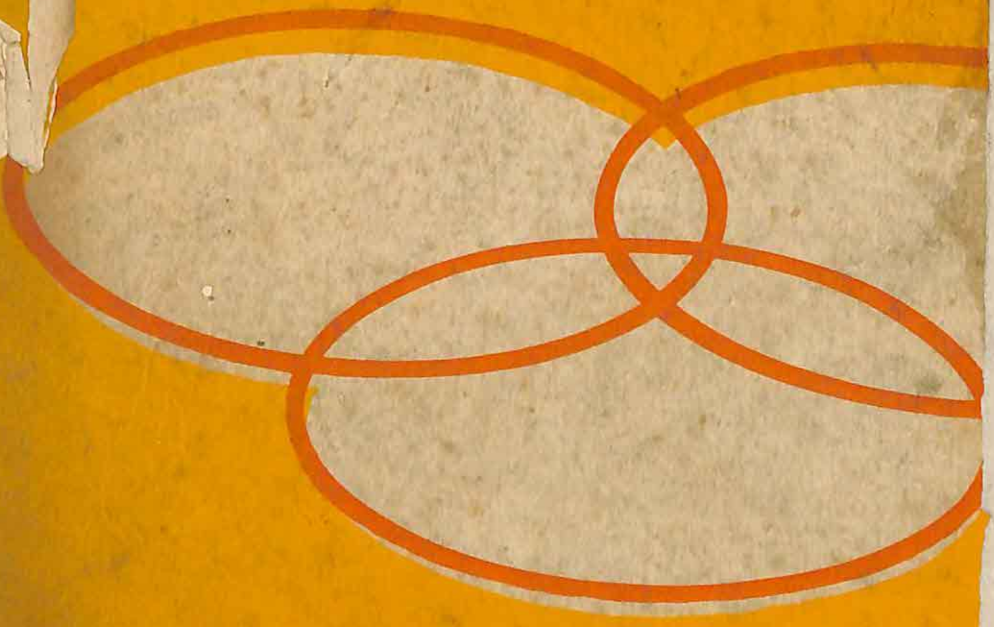

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